

Referee's report on the habilitation thesis

Thesis author: Dr. rer. nat. Hana Šmitala Mizerová

Thesis title: Lax equivalence principle in compressible fluid dynamics

1. Relevance of the topic

The thesis is of relevance to the field of numerical mathematics, advancing the understanding and analysis of compressible fluid dynamics models within the framework of measure-valued solutions.

2. Original contribution

The research focuses on the development of analytical approaches for studying the convergence of numerical approximations of solutions to models of unsteady compressible fluid flow. A central aspect is the design of a new convergence strategy for nonlinear conservation laws in fluid dynamics. This approach is motivated by the classical Lax–Richtmyer equivalence principle, according to which stability and consistency imply, or are equivalent to, the convergence of a numerical scheme. In its generalized form, developed in collaboration with E. Feireisl, M. Lukáčová and B. She, this approach enables a rigorous analysis of numerical methods for the Euler, Navier–Stokes and Navier–Stokes–Fourier models of compressible fluid flows.

The introductory part of the thesis is synthetic in nature and provides an exposition of the main ideas and scientific results achieved by the author and her co-authors. It also includes selected, appropriately adapted sections from their joint published works. The core of the thesis consists of a selection of six scientific articles published in high-ranking peer-reviewed journals.

Feireisl, Lukáčová-Medvid'ová and Mizerová (2020a) study entropy-stable finite-volume schemes for compressible inviscid fluids, proving strong convergence. Consistency for the complete Euler system requires positive density and bounded mechanical energy, while for the barotropic case ($p = p(\rho)$) only positive density is needed.

In Feireisl, Lukáčová-Medvid'ová and Mizerová (2020b), a finite-volume scheme is proposed, motivated by the viscous regularization of the complete Euler system introduced by Guermond and Popov (2014). The convergence was investigated within the class of dissipative measure-valued solutions.

Mizerová and She (2020) prove unconditional convergence of a staggered-grid finite-difference scheme for the multi-dimensional compressible Navier–Stokes equations to a strong solution for the adiabatic exponent $\gamma > 1$, using dissipative measure-valued solutions and weak–strong uniqueness. For $\gamma > d/2$, they obtain uniform error estimates with explicit rates, confirmed by numerical experiments.

Papers by Basarić, Lukáčová-Medvid'ová, Mizerová, She and Yuan (2023) and by Basarić, Feireisl and Mizerová (2023) address regularity and convergence for the compressible Navier–Stokes–Fourier system, but from complementary perspectives. Both papers rely on conditional regularity, using boundedness of physical quantities to control solutions. The former applies this philosophy to numerical analysis, proving convergence and error estimates for a fully discrete finite volume scheme under bounds on density and temperature. The latter is analytical, showing that strong solutions with Dirichlet boundary conditions can only break down if density, temperature, or velocity blow up.

Finally, the paper by Mizerová (2025) establishes dissipative-measure-valued-strong uniqueness for 3D compressible MHD flows, both conditionally (under specific assumptions on the transport parameters

and thermodynamic constitutive relations) and unconditionally. This framework ensures that stable, consistent numerical approximations converge reliably to strong solutions of the MHD system.

3. Comments

The above-mentioned works were subject to a peer-review process prior to their publication, and I will therefore not re-evaluate them. The text of the habilitation thesis itself is concise, yet specific and sufficient. I have the following specific comments for the author to consider during the defense:

1. For linear problems, the Lax principle provides a clear roadmap for numerical analysis: (i) verify consistency (often straightforward); (ii) establish stability, (typically the main analytical challenge); and (iii) deduce convergence automatically. For nonlinear problems (e.g. nonlinear conservation laws, compressible fluid dynamics), the principle does not hold verbatim, but variants of the Lax philosophy still guide convergence analyses toward weak or measure-valued solutions.

In such nonlinear settings, what is typically the main analytical challenge, and how can this challenge be *briefly* illustrated in the simpler case of a barotropic Euler system?

2. In Feireisl, Lukáčová-Medvid'ová, Mizerová (2020b), the Brenner (2005, 2006) model for compressible *viscous* fluids is employed to develop a new finite-volume scheme for the compressible *inviscid* fluids. The essence of Brenner's approach is a dual-velocity framework that separates the motion of mass (mass velocity $\mathbf{v}_m$) from the transport of fluid volume (volume velocity $\mathbf{v}_v$). By introducing two velocities, the standard balance equations are modified (e.g. mass conservation now depends on $\mathbf{v}_m$, momentum balance uses $\mathbf{v}_v$ to account for the microscopic motion contribution to stress, and energy balance includes the energy carried by microscopic diffusion). This leads to a modified Navier–Stokes system with additional terms representing diffusive fluxes of mass, momentum, and energy.

What motivates using a viscous regularization based on Brenner's dual-velocity model to study the *classical* compressible Euler equations? One would expect the classical single-velocity Navier–Stokes model to be sufficient.

4. Conclusion

The submitted habilitation thesis meets the requirements for this type of work in terms of topicality, scientific content, scope and form of its presentation. The results achieved have an appropriate impact and expand the level of knowledge in the field of mathematical fluid dynamics. I recommend accepting the habilitation thesis and awarding Dr. rer. nat. Hana Šmitalová Mizerová the academic title of Associate Professor in the study field of Mathematics.

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