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Review of the habilitation of Tibor Macko

Surgery theory has been developed with great success to classify high-dimensional manifolds. It is an enormous theory whose development began over 50 years ago. Very roughly it translates questions about manifolds and their automorphisms into questions in algebra, in particular in K- and L-theory. Solving the resulting questions in K- and L-theory typically remains difficult, but can be done in many cases and then leads to concrete and satisfying geometric results. Tibor Macko is one of the leading experts for this theory. He has contributed both to the development of the theory and to the applications of the theory through calculations. Some of these contributions are the content of the present habilitation thesis. Before discussing the thesis below, I want to comment on a different aspect of Macko's work. He has also made important and very welcome contributions by reworking the theory. For example, his paper with Kühl and Mole on Ranicki's total surgery obstruction reworked and clarified this central obstruction. Before Macko's paper the construction was spread out over two books and a number of papers, and not very accessible, even for the leading experts in surgery theory. I expect that Macko's book project with Crowley and Lück on surgery theory will be important for the further development of surgery theory. Surgery theory is certainly in need of foundational literature.

The thesis begins with an introduction to surgery theory. On the one hand this introduction serves as a very accessible introduction to surgery theory, on the other hand it sets the stage very nicely for the reader to appreciate the papers of Macko selected for the habilitation thesis.

The first four papers concern the structure sets of lens spaces. Lens spaces are quotients of unitary actions of cyclic groups on spheres. More generally, fake lens spaces are quotients of free actions of cyclic groups on spheres. Lens spaces have been important examples from the beginning of surgery theory, because they provided the first examples of pairs of manifolds that are homotopy equivalent, but not homeomorphic. The main result of the paper [A1] is the computation of the topological structure set of lens spaces with fundamental group cyclic of order a power of 2. This is joint with Wegner. This set organizes the collection of manifolds simple homotopy equivalent to a given lens spaces into an abelian group. The computation of this abelian group proceeds through the surgery exact sequence. The relevant L-group have been computed via the G-signature theorem in terms of representation theory, in particular by Hambleton-Taylor. To obtain a calculation of the structure set from this the ρ-invariant is crucial in [A1]. It is defined using the G-signatures of manifolds bounding a sum of the given lens space. A priori, it is not even clear that this invariant defines a group homomorphisms, but one of the results of [A1] is that it is indeed a group homomorphism. The main step in [A1] is to understand the kernel of the ρ-invariant. This is an intricate and difficult calculation. The paper [A3] gives a full summary of the classification of fake lens spaces. The case where the cyclic group is of odd order is due to Wall. The case where the group is of even order is due to Macko and Wegner, see [A1, A3]. In surgery the structure set of manifold M has a generalization to the structure space of M. The set (or group) of components of the structure space is then the structure set. Its higher homotopy groups are structure sets of products $M \times D^k$. In [A4, A5] the results for the structure sets of lens spaces are extended to structure spaces. This proceeds analogous to the results in [A1, A2]. An important new ingredient is a new formula for the ρ-invariant defined on the structure set of $\mathbb{C}P^d \times D^{2k}$.

It is a puzzling outcome of surgery theory that the topological structure set $\mathscr{S}(M)$ of a manifold M is an abelian group. Elements of the structure set are equivalence classes of homotopy equivalences $f: N \to M$ and it is not at all clear how given two homotopy equivalences to M one can construct a third homotopy equivalence to M. This is much more clear for the relative structure sets $\mathscr{S}_\partial(M \times D^k)$; simple relative homotopy equivalences to $M \times D^k$ can be stacked, using a chosen coordinate in D^k. Periodicity (or more precisely near-periodicity) states that there is a short exact sequence

$$\mathscr{S}(M) \to \mathscr{S}_\partial(M \times D^k) \to H_0(M;\mathbb{Z}).$$

Thus the problem of understanding the group structure on the structure set geometrically is closely related to understanding the periodicity map geometrically. Building on work of Cappell–Weinberger and of Hutt a geometric understanding of the periodicity map is achieved in [B1] and used to show that the ρ-invariants is in general a group homomorphism (and not just for lens spaces). This an important foundational result that already found applications, for example in work of Davis and Lück.

Informally one often states that the structure set $\mathscr{S}(M)$ classifies manifolds simple-homotopy equivalent to M. But this is not quite true; elements of the structure set are really manifolds N together with a choice of homotopy equivalence $f: N \to M$. Classifying manifolds that are simple-homotopy equivalent to M without a choice of simple-homotopy equivalence amounts to understanding the quotient of $\mathscr{S}(M)$ by the action of the group of simple self-homotopy equivalences $G(M)$. Of course, usually understanding $G(M)$ is a difficult task, but every s-cobordism invariant induces an invariant on $\mathscr{S}(M)$ that factors over the quotient. In [B4] Macko (joint with Crowley) studies the relation between the structure set $\mathscr{S}(M)$ and its quotient by $G(M)$. The main result uses denominators in Hirzebruch's $\mathscr{L}$-polynomials to show that in many cases (for example for simply connected manifolds) $\mathscr{S}(M)$ is infinite if and only if the quotient is infinite. On the other hand the paper also gives examples where $\mathscr{S}(M)$ is infinite, but the action of $G(M)$ is transitive.

The final paper of this habilitation thesis concerns Nil-terms in A-theory. A-theory has been developed by Waldhausen and relates information contained in structure spaces of manifolds as they appear in surgery theory, to spaces of automorphisms of manifolds via the parametrized h-cobordism theorem. The paper [C1] concerns Nil-terms in this theory. Nil-terms appear in the computation of $A(X \times S^{-1})$, and are non-linear analogs of the Nil-terms appearing in the Bass–Heller–Swan decomposition of the K-theory of Laurent polynomial rings $R[t,t^{-1}]$. In the linear situation of Bass–Heller–Swan there is additional structure present on the Nil-terms that can be used to show that they are typically quite large. The paper [C1], joint with Grunewald and Klein extends this structure to the A-theoretic Nil-terms and uses it to deduce that the p-primary subgroups of their homotopy groups are not finitely generated (unless they are trivial). In addition trace methods are used to compute the A-theoretic Nil-terms in a range of degrees. Via work of Farrell–Jones this result leads to non-trivial families of automorphisms of manifolds of negative curvature.

I am very impressed by Macko's achievements in surgery theory and the range of his results. I wholeheartedly recommend his habilitation thesis for acceptance and that the title "Docent" should be awarded to Tibor Macko.

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