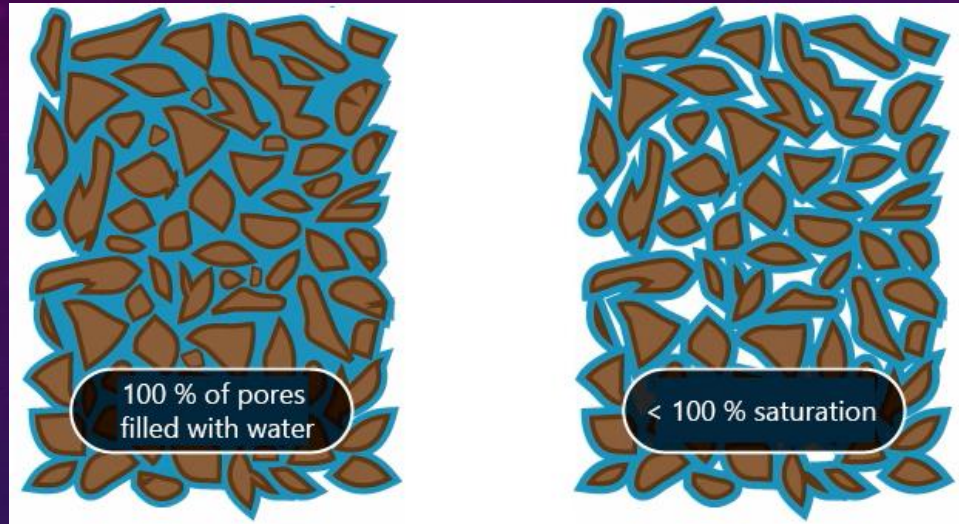


The background features a dark blue gradient with a subtle pattern of white dots. Overlaid on the left side are several concentric circular patterns. A prominent circular scale with tick marks and numerical labels (40, 150, 160, 170, 180, 190, 200, 210, 220, 230, 240, 250, 260) is visible. Other circular elements include dashed lines, solid lines, and arrows indicating clockwise or counter-clockwise rotation.

WATER FLOW IN POROUS MEDIUM

(BASIC CONCEPTS)

WATER MOVEMENT IN POROUS MEDIUM (two basic concepts/scenarios)



1. Saturated medium (100% of pore space is filled with water)

Darcy (1856) showed that the saturated flow (conductivity) through porous medium is linear with the gradient in potential. This is quite different compared to falling (solid/liquid) body through air atmosphere (or bike/car driving along the street), where air resistance is proportional to the velocity squared.

$$q = -K_s \frac{\Delta H}{L}$$

2. Unsaturated medium (< 100% of pore space is filled with water)

Water flow (conductivity) is a function of water content. Hydraulic conductivity of certain porous medium increases (nonlinearly) as its water content increases. Why? As the moisture (saturation) increases, number of “routes” increases. On the contrary, when moisture level is lower, not only there are less routes available, but the trajectories are longer. Lowering the water content leads to narrowing of the cross-sectional area available for water flow. Nonlinear increase in conductivity of porous medium with increase in water content also follows from Poiseuille’s law, according to which conductivity of a pore is proportional to 4th power of its radius.

Buckingham modified Darcy’s equation to express water flow in unsaturated conditions as a function of matric and gravity potentials

$$q = -K(h) \frac{\partial(h+z)}{\partial z} = -K(h) \left(\frac{\partial h}{\partial z} + \frac{\partial z}{\partial z} \right) = -K(h) \left(\frac{\partial h}{\partial z} + 1 \right)$$

$$q = -K(\psi_m) \left[\frac{\partial \psi_m}{\partial z} + \frac{\partial \psi_z}{\partial z} \right]$$

$$q = -K(\psi_m) \left[\frac{\partial \psi_m}{\partial z} + 1 \right]$$

WATER MOVEMENT IN NARROW TUBES

Simple scenario - water movement in narrow tubes: water exhibits certain viscosity, which means that there is an attraction between adjacent layers of moving water. These are so called tangential forces that are causing shear stress among moving parts of water. Additionally, water does partially adhere to the surface of pores, which means that water directly contacting the pore walls moves slower in comparison to water molecules that are located at certain distance from the surface.



WATER MOVEMENT IN NARROW TUBES

Maintaining the relative motion of the plates at a certain velocity requires applying a tangential force to overcome the frictional resistance in the fluid. This resistance, per unit area of the plate, is proportional to the velocity of the upper plate U and inversely proportional to the distance h between the plates. The **shearing stress** (τ_s) at any point is proportional to the velocity gradient du/dy . The **viscosity** η is the proportionality factor between τ_s and du/dy :

$$\tau_s = \eta \, du/dy$$

In the pipe the velocity of moving water is lowest at the wall and highest at the pore's axis. Fluid flow in a horizontal tube is normally caused by a pressure gradient acting in the axial direction. A fluid “particle” is accelerated by the pressure gradient and retarded by the frictional resistance.

WATER MOVEMENT IN NARROW TUBES

There are two forces acting:

- **Pressure** acting on the face of the cylinder: $\Delta p \pi y^2$ (where $\Delta p = p_1 - p_2$)
- **Frictional resistance** due to shear force acting on the circumferential area: $2\pi y L \tau_s$

When pressure gradient equals frictional resistance, water moves with constant velocity (there is no acceleration).

Then shearing stress $\tau_s = (\Delta p / L)(y / 2)$

Recalling equation $\tau_s = -\eta (du/dy)$ (the negative sign arises because in this case u decreases with increasing y), we obtain $du/dy = -(\Delta p / \eta L)(y / 2)$

The velocity of the water movement in the pipe (column) is given by:

$$u(y) = (\Delta p / 4\eta L)(R^2 - y^2)$$

WATER MOVEMENT IN NARROW TUBES

The previous equation indicates that the velocity is distributed parabolically over the radius, with maximum velocity on the axis ($y = 0$): $u_{\max} = \Delta p R^2 / 4\eta L$

To obtain the discharge Q (the volume flowing through a section of length L per unit time), we have to calculate the volume of a circular paraboloid, which is given by:

$$Q = (1/2)\pi R^2 u_{\max} = \pi R^4 \Delta p / 8\eta L$$

This equation, known as **Poiseuille's law**, indicates that the volume flow rate is proportional to the pressure drop per unit distance ($\Delta p/L$) and the fourth power of the radius of the tube.

The mean velocity over the cross section is

$$u_{\text{mean}} = \Delta p R^2 / 8\eta L = (R^2 / a\eta) \nabla p$$

where ∇p is the pressure gradient ($\Delta p/L$). Parameter a , equals to 8 in a circular tube, varies with the shape of the conducting passage.

Example 1

The water in an irrigation hose is at a hydrostatic pressure of 100 kPa (1 bar). Five drip irrigation emitters are inserted into the wall of the hose. Calculate the drip rate (L/hr) from the emitters if each contained a coiled capillary tube 1 m long and the capillary diameters are 0.2, 0.4, 0.6, 0.8, and 1.0 mm. Assuming laminar flow, what fraction of the total discharge is due to the single largest emitter?

We use Poiseuille's law $Q = \pi R^4 \Delta P / 8 \eta L$ ($100 \text{ kPa} = 1 \text{ bar} = 10^5 \text{ N/m}^2 = 10^5 \text{ kg/m sec}^2$).

$$Q_1 = 3.14 \times (0.0001)^4 \times 100000 / 8 \times 0.001 \times 1 = 3.925 \times 10^{-9} \text{ m}^3/\text{s} = 0.01413 \text{ L/h}$$

$$Q_2 = 0.22608 \text{ L/h}$$

$$Q_3 = 1.14453 \text{ L/h}$$

$$Q_4 = 3.61728 \text{ L/h}$$

$$Q_5 = 8.83125 \text{ L/h}$$

$$Q_{\text{total}} = 13.83 \text{ L/h and the } Q_5 \text{ is } 63.84 \% \text{ of } Q_{\text{total}}$$

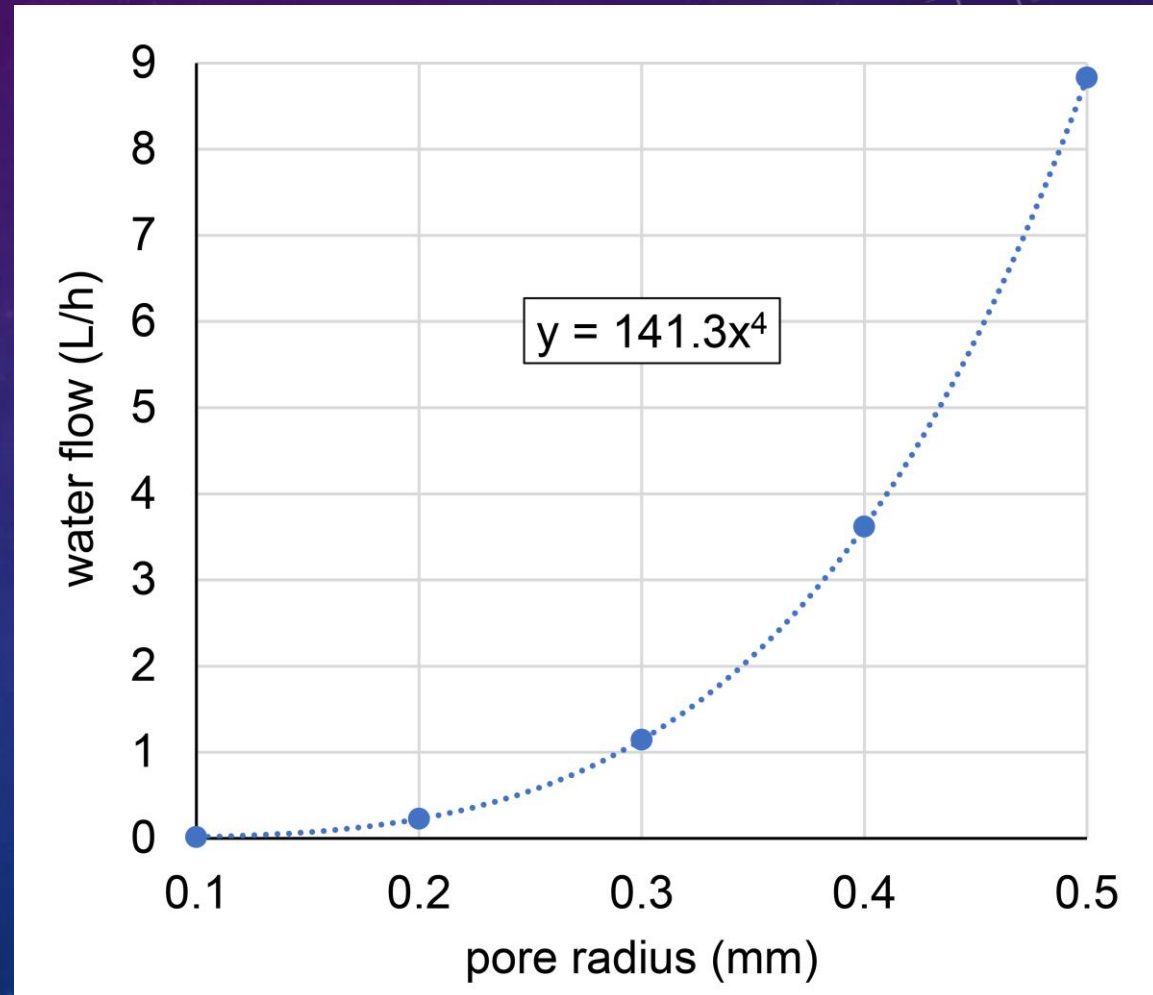
Water flow (calculated using Poiseuille's law) in narrow pipe (tube) depicted as function of pore radius

this shows that hydraulic conductivity of a pore is proportional to 4th power of its radius

when we compare for example, 1 mm pore and 0.1 mm, the hydraulic resistance of later pore will be 10 000 times higher compared to former

this doesn't mean however that medium (soil) with 1 mm pores will show 10 000 times higher conductivity compared to soil with 0.1 mm pores; K values will be "only" 100 times higher; why? because effect of pore radius size will be compensated by higher pore abundance in considered volume of medium

note: flow is affected obviously also by applied pressure, but if pressure is constant, flow is controlled by the pore size



WATER MOVEMENT IN SATURATED POROUS MEDIUM

- flow Q : the volume (V) moving through the column per unit time (t);
- hydraulic gradient: the head drop (ΔH) per unit distance (L) in the direction of flow;
- flux density (or simply just “flux”, q): the volume of water flowing through a unit cross-sectional area per unit time (t);
- hydraulic conductivity of porous medium, K is the velocity of water flow

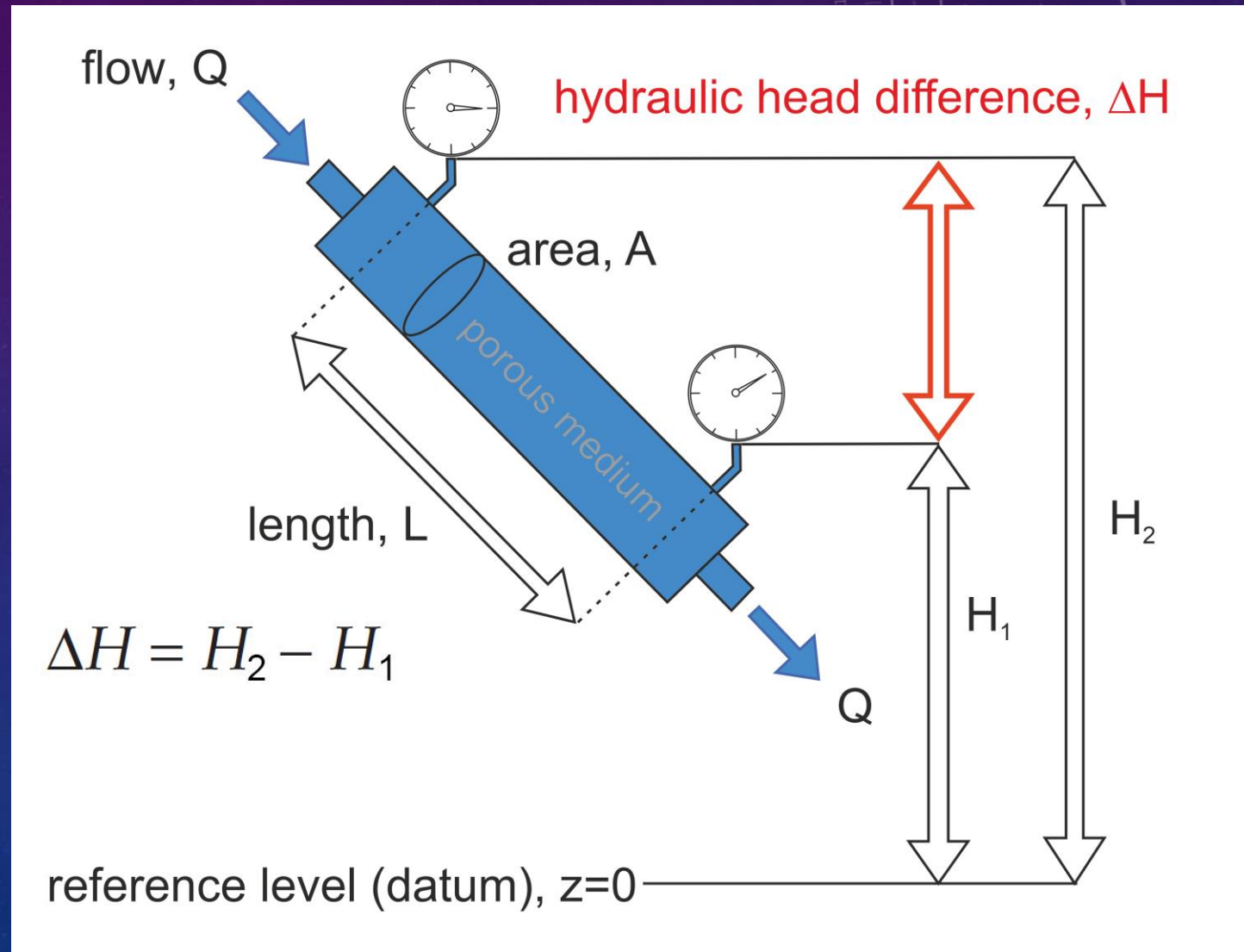
Darcy's law:

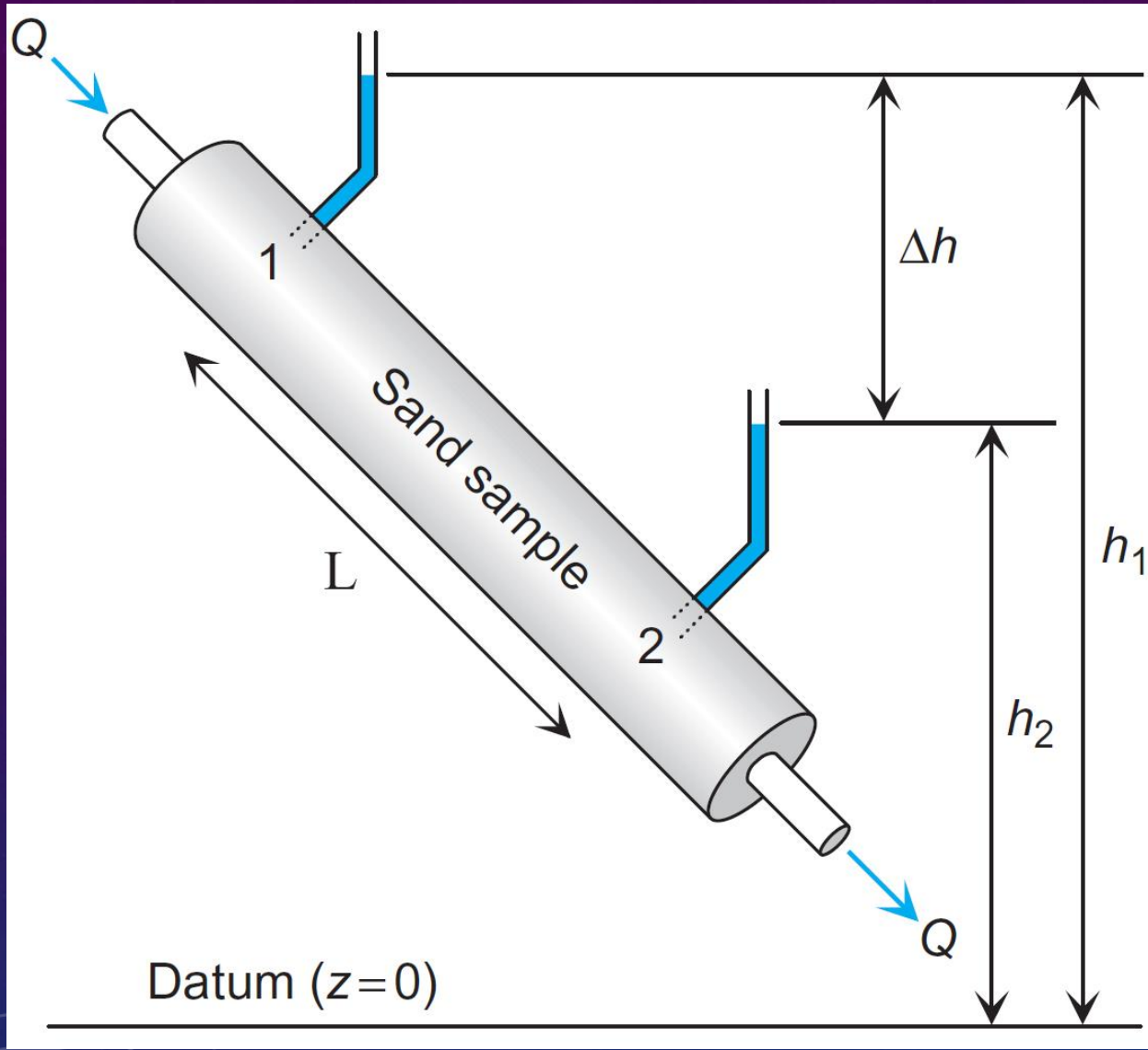
$$q = -K \Delta H / L$$

$$q = Q/A = V/At \propto \Delta H/L$$

proportional

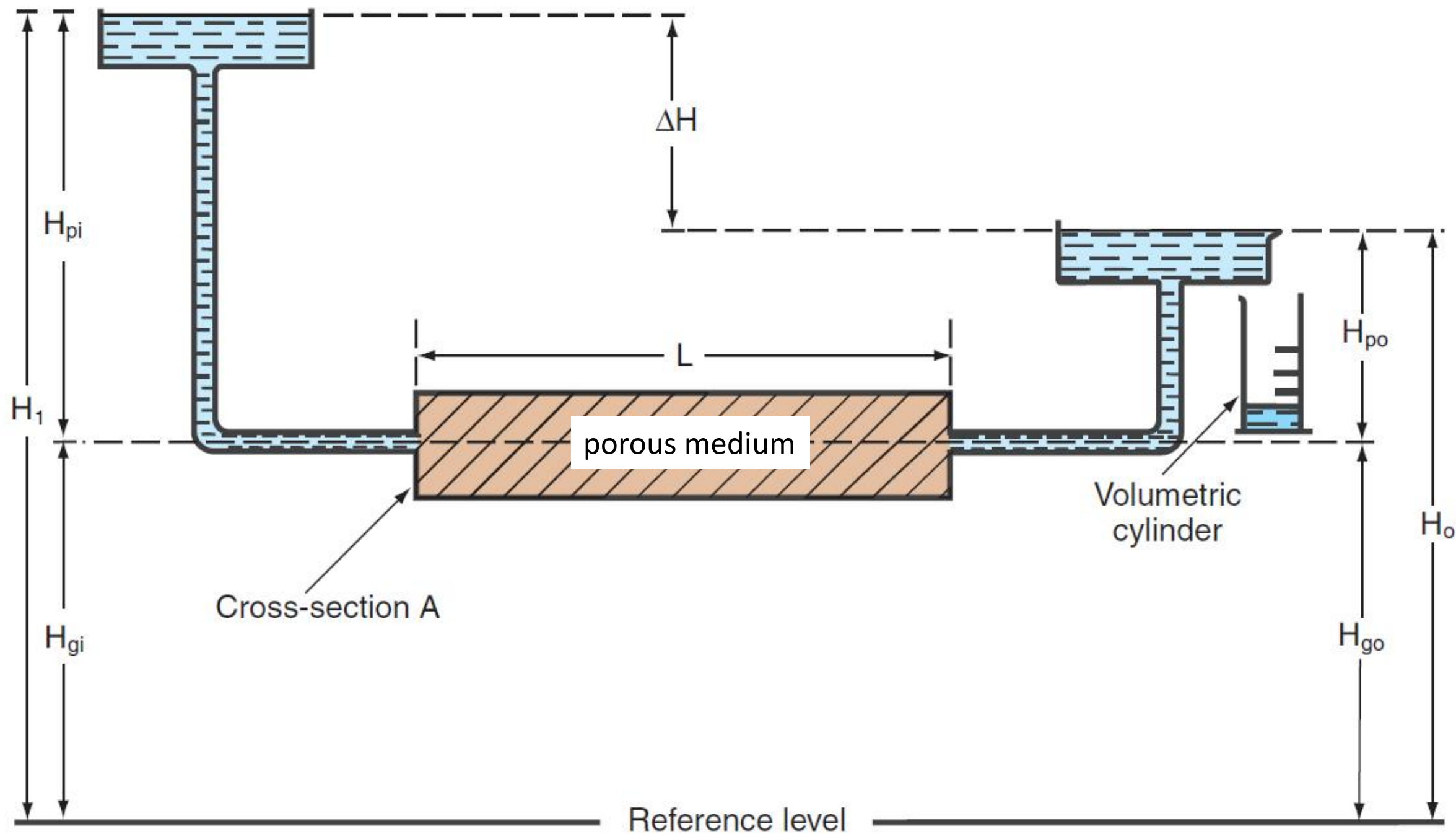
$$Q = V/t \propto A \Delta H / L$$





Example 2: A sample of silty sand is tested in a laboratory experiment. The column has an inside diameter of 10 cm and the length between manometers is $L = 25$ cm. With a steady flow of $Q = 1.7 \text{ cm}^3/\text{min}$, the head difference between the manometers is $\Delta h = 15$ cm. Calculate the hydraulic conductivity K_s .

$$\begin{aligned}
 K_s &= -Q_s \frac{1}{A} \frac{L}{\Delta h} \\
 &= -1.7 \text{ cm}^3/\text{min} \left(\frac{1}{\pi (5 \text{ cm})^2} \right) \left(\frac{25 \text{ cm}}{-15 \text{ cm}} \right) \\
 &= 0.036 \text{ cm/min} \\
 &= 6.0 \times 10^{-4} \text{ cm/sec}
 \end{aligned}$$



Example 3

Purifying Dijon's water using sand filter beds. Dijon has app. 10000 citizens, and each consumes 20 liters of water per day (on average). Let's suppose that we know that 30 cm of sand column is adequate for filtration and that the sand that is available has hydraulic conductivity of 0.00002 m/s. Calculate the area of the filter bed needed under a hydrostatic pressure head of 0.7 m.

1. we will calculate the discharge Q needed per day:

$$10000 \times 20 \times 0.01 / 86400 = 0.00231 \text{ m}^3 \text{ per second}$$

2. Darcy's law $Q = AK\Delta H/L \rightarrow A=QL/K\Delta H$

hydraulic head drop $\Delta H = 0.7 + 0.3 = 1 \text{ m}$

$$A = (0.00231 \times 0.3) / 0.00002 = 34.65 \text{ m}^2$$

note: per capita water use in the U.S. (with running toilets, showers, dishwashers, etc.) ranges from 100 to 400 L/day

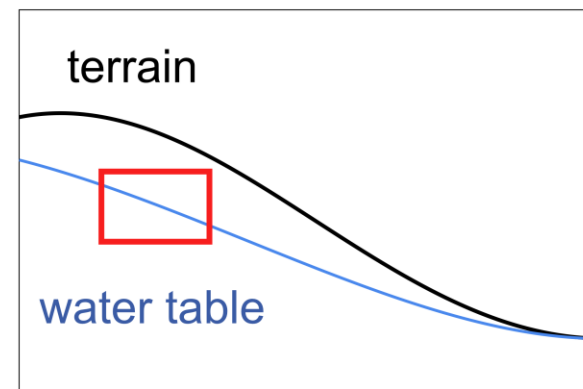
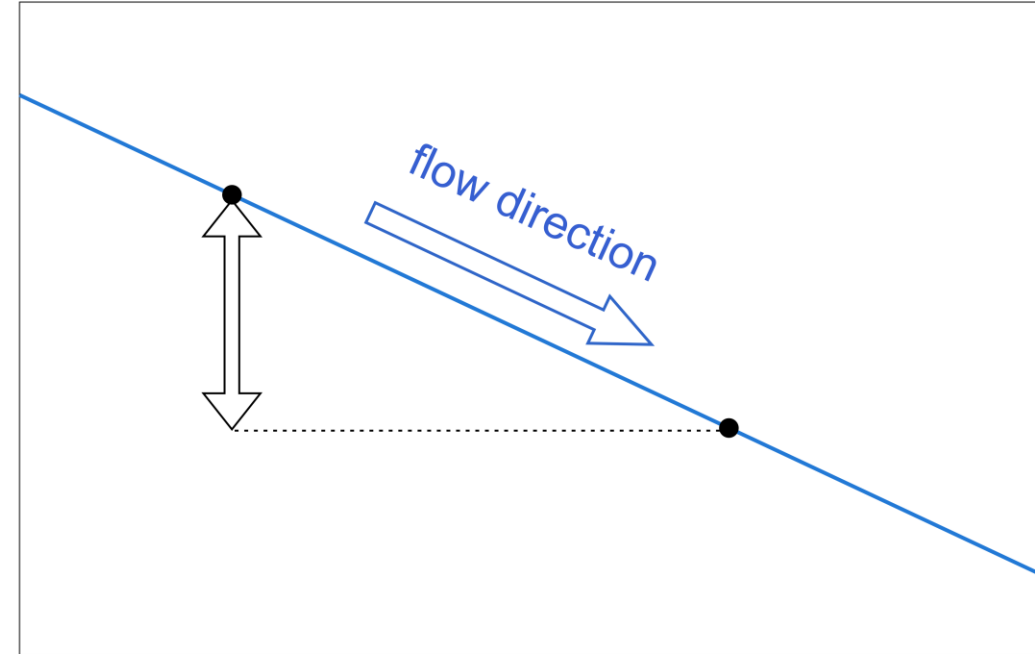
WATER MOVEMENT IN SATURATED POROUS MEDIUM

Total hydraulic head in porous environment is composed mainly of two components:

1. pressure component
2. gravity component
3. velocity component (is usually neglected due to very small velocity of water moving in the subsurface)

the two displayed points differ in gravity head but their pressure head is the same

the flow will be driven by difference in gravity head



Example 4

Let's say we have steady downward flow through a two-layer porous profile, the top of which is submerged under a 1 m head of water and the bottom of which is defined by a water table. Each layer is 0.5 m thick. The K values for the two layers are 10^{-6} m/s and 10^{-7} m/s respectively. Calculate the flux, hydraulic and pressure heads at the interface between the layers for the two scenarios: 1. layer with lower conductivity will be located above the layer with higher K value, 2. reverse order of layers. We use the Ohm's law analogy for steady flow through two resistors in series:

$$q = \Delta H / (R_1 + R_2),$$

where q is the flux, ΔH is the total hydraulic head drop across the profile, and R_1, R_2 are the hydraulic resistances of the top layer and sublayer, respectively. The resistance is proportional to the layer's thickness and inversely to its conductivity ($R = L/K$).

$$q = (1 \text{ m} + 1 \text{ m}) / (0.5 \text{ m} / 10^{-6} \text{ m/s} + 0.5 \text{ m} / 10^{-7} \text{ m/s}) = 3.64 \times 10^{-7} \text{ m/s}$$

If we consider the top layer alone, the hydraulic head at the interlayer interface can be calculated according to:

$$q = K_1 \Delta H_1 / L_1 = K_1 (H_{\text{surface}} - H_{\text{interface}}) / L_1 \rightarrow H_{\text{interface}} = H_{\text{surface}} - q L_1 / K_1 = 2 \text{ m} - (3.64 \times 10^{-7} \text{ m/s})(0.5 \text{ m}) / 10^{-6} \text{ m/s} = 1.818 \text{ m}$$

Since the gravity head at the interface is 0.5 m (above our reference datum at the bottom of the profile), the pressure head H_p is

$$H_p = H - H_g = 1.818 - 0.50 = 1.318 \text{ m}$$

Example 4

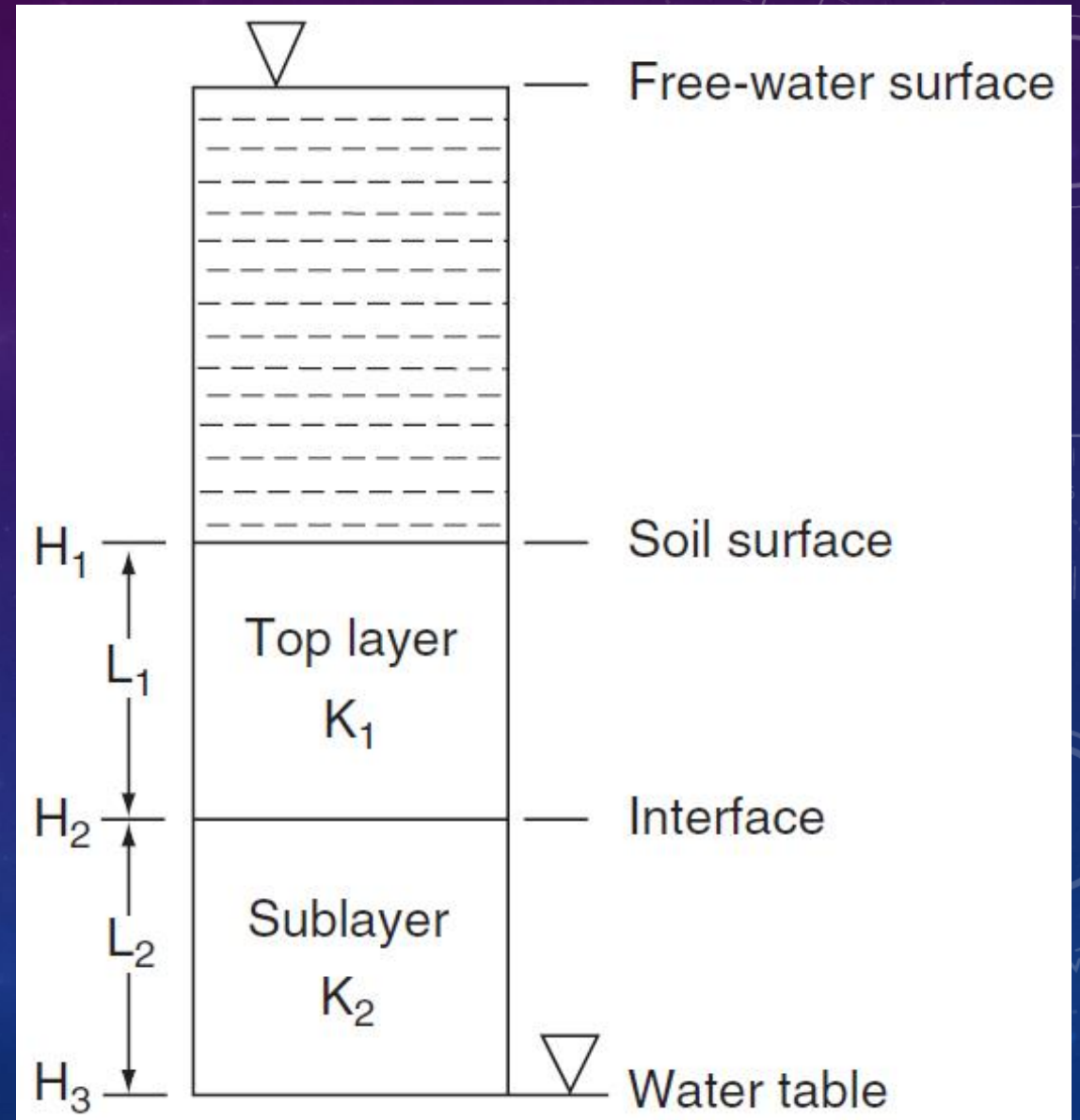
After reversing the order of the layers (less conductive layer overlays more conductive,) total head drop remains the same and so does total resistance. Therefore, the flux remains the same as well.

$$H_{\text{interface}} = 2.0 - (3.64 \times 10^{-7})(0.5)/10^{-7} = 0.18 \text{ m}$$

The pressure head at the interface will be

$$H_p = H - H_g = 0.18 - 0.50 = -0.32 \text{ m}$$

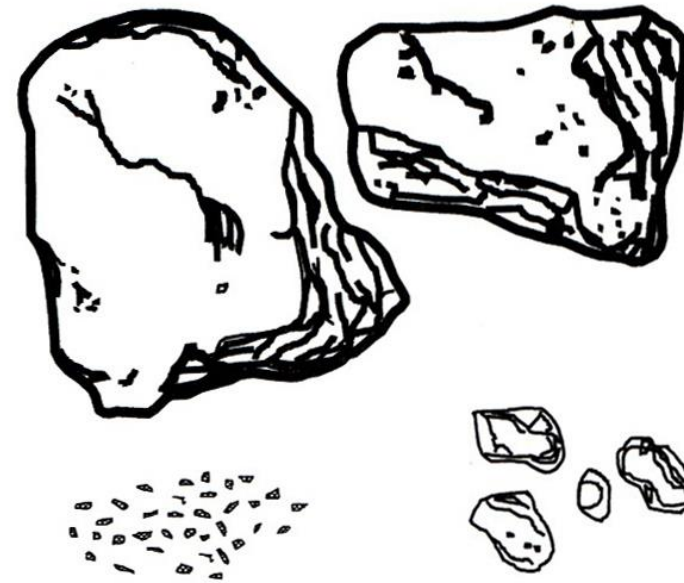
With the more conductive layer on top, flow is impeded at the interface and there is a pressure buildup. The opposite occurs when the upper layer is less conductive.



Approximate hydraulic conductivities (K_s) and capillary rise values (h_c) of porous medium made of particles size fractions

sand (2 – 0.05 mm)

clay (< 0.002 mm)



100 μm = 0,1 mm

silt (0.05 – 0.002 mm)

Hydraulic Conductivity at Saturation (K_s), Particle Diameters (d), and Height of Capillary Rise (h_c) for some Solids^a

	K_s (m/sec)	d (mm)	h_c (m)
Clay	10^{-10} – 10^{-8}	<0.002	2–4
Silt	10^{-8} – 10^{-6}	0.002–0.05	0.7–1.5
Sand	10^{-5} – 10^{-3}	0.05–2	0.12–0.35
Gravel	10^{-2} – 10^{-1}	>2	Nil

HYDROLOGICAL MODELS

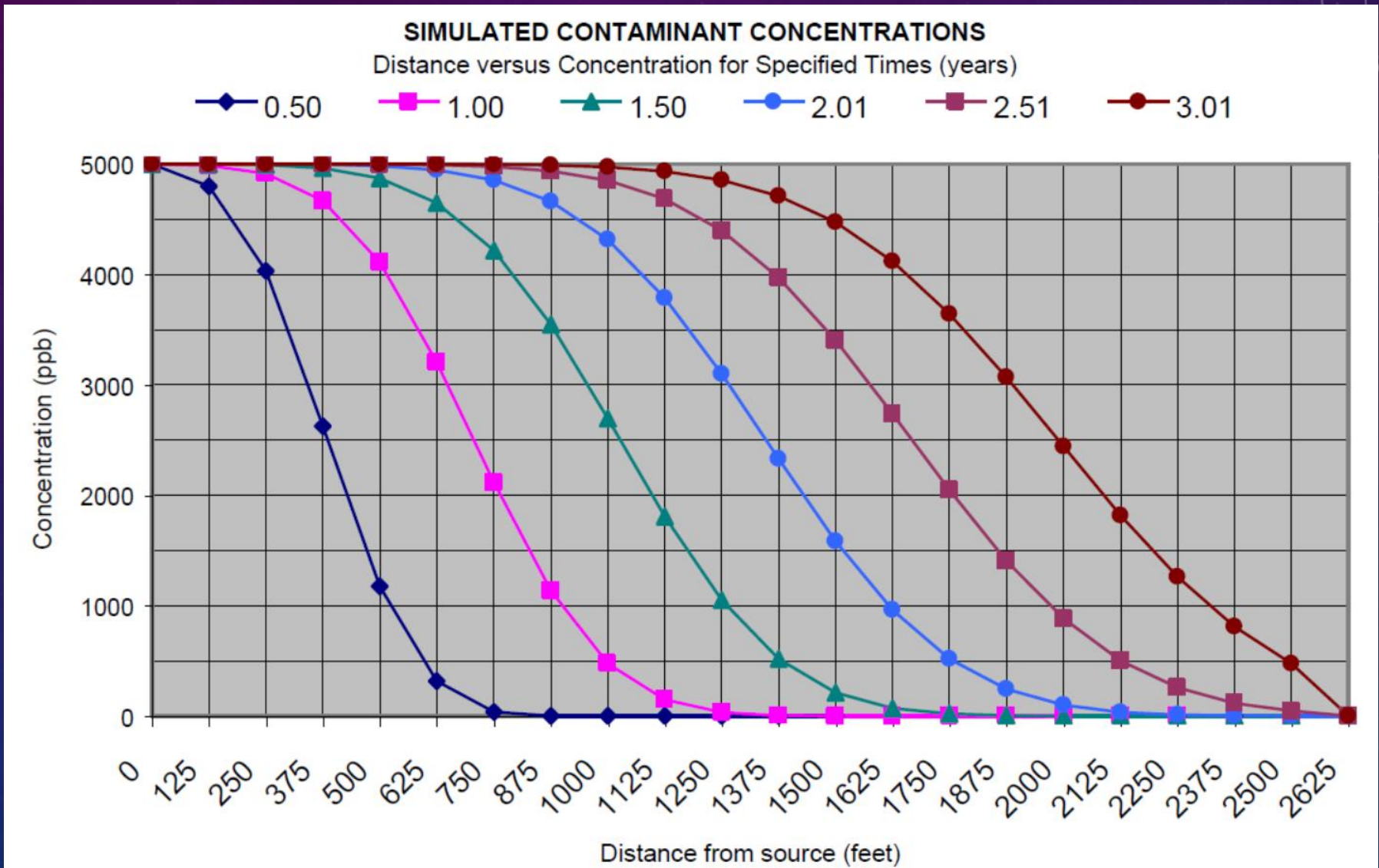
- hydrological models enable to assess quantitatively the water movement in the environment
- among them are the advection-dispersion models that can be used (for example) if we want to predict how the contaminant is spreading in the subsurface through groundwater movement
- advection is mechanical transport of solutes along with the bulk flux of the water (similarly as in case of Darcy's law), whereas dispersion refers to hydromechanical mixing, caused by the detours in the trajectory of moving water (additionally mixing of water solution is caused partially also by diffusion, which however plays minor/insignificant role considering larger scales)
- combining the two processes into single formula gives us advection-dispersion (AD) equation

HYDROLOGICAL MODELS (EXAMPLE 4)

- let's assume that we want to predict (estimate) how fast the contaminant in ground water is spreading in certain direction → numerical model developed by E. Scott Bair (Ohio State University)
- the model “The Woburn 1-D Contaminant Transport Game” is an excel spreadsheet that enables to compute the concentration of TCE (Trichloroethylene) traveling in the groundwater flow system toward the river (well H) from the source of contamination (W.R. Grace)
- the spreadsheet computes values of hydraulic head, advective flow velocities and travel times, contaminant velocities and travel times, and contaminant concentrations at 20 locations along the flowpath
- the outputs from the model can be visualized by breakthrough curves showing changes in concentration versus distance and changes in concentration versus time

HYDROLOGICAL MODELS (EXAMPLE 4)

feet x 0.3048 = meters
125 feet ≈ 38 m
2625 feet ≈ 800 m



HYDROLOGICAL MODELS (EXAMPLE 4)

- hydrogeologic input parameters for the model are porosity (n), aquifer thickness (b), hydraulic conductivity (K), and recharge rate in the 20 cells along the flowpath; input values include also the initial concentration of the contaminant at point source (C_0), time step duration (Δt), longitudinal dispersivity (α), and chemical retardation (R_f)
- the concentration (C_i) for each step (in time and space) is calculated according to equation:

$$C_i^t = C_i^{t-1} + \frac{\Delta t}{R_f \Delta x} \left(\frac{\alpha_x v_{i+1/2} (C_{i+1}^{t-1} - C_i^{t-1}) - \alpha_x v_{i-1/2} (C_i^{t-1} - C_{i-1}^{t-1})}{\Delta x} \right) - \frac{\Delta t}{R_f \Delta x} (v_{i+1/2} C_i^{t-1} - v_{i-1/2} C_{i-1}^{t-1})$$

- where $\alpha_x v_i = D$ is hydrodynamic dispersion (molecular diffusion is ignored) and the terms $v_{i-1/2}$ and $v_{i+1/2}$ are flow velocities at the interfaces between neighboring cells i and i-1 and cells i and i+1, respectively

HYDROLOGICAL MODELS (EXAMPLE 4)

alternatively, the model's output may be visualized as a 3D surface, expressing spatiotemporal changes in contaminant concentration in groundwater

